Tools Documentation

March 2017

Overview

Classroom Learning Partner (CLP) tools allow students and teachers to create, annotate, and manipulate visual representations to solve math problems. The tools may be used for a number of mathematical purposes, but were mainly conceived to assist in creating visual representations for multiplication and division.

The underlying model of multiplication and division assumed by the current set of tools involves a repetition of groups of the same size. Many of the tools may be used for either multiplication *or* division but some are more natural for one than the other, as reflected in the tool recommendations for each operation below.

The tools may also be used for both partitive and quotative division, though some tools may be more easily imaginable with one type of division than the other. Both types of division are described below.

Table of Contents

Tools Overview

CLP contains seven tools for use in creating visual representations, as shown below in these examples. More details about using each tool follow these examples.

Multiplication Tools

Stamp, Number line, Array

Ana has 3 baskets. Each basket has 5 apples in it. How many apples are in Ana's baskets?

Stamp

Number line

Array

Division Tools

Division group, Bin, Number line, Array, Division template

Pablo has 20 fish in fish tanks. He has 4 tanks. Each tank has the same number of fish in it. How many fish are in each tank?

Division Group

Bin

Number line

Array

Division Template

Two Types of Division

The two types of division are partitive and quotative division, where you either know the the number of groups or the number in each group. Word problems are always one or the other because they explicitly specify this information. Since non-word problems (e.g., 114÷12) don't specify what the numbers refer to, students may interpret them either way.

Partitive Division

You know the number of groups, and you are trying to find the number in each group, e.g., *Pablo has 20 fish in fish tanks. He has 4 tanks. Each tank has the same number of fish in it. How many fish are in each tank?*

Quotative Division

You know the number in each group, and you are trying to find the number of groups, e.g., *Paola has 20 fish in fish tanks. Each tank has 5 fish in it. How many tanks are there?*

How-To (All Tools)

Watch the 5-minute Introduction to CLP video before using the tools: https://youtu.be/qCyjuUlGgqY (Also under the **CLP Software** webpage of the CLP website.) The video explains how to create visual representations using a finger or tablet pen for drawing and a Tool Menu for adding objects, such as arrays and number lines.

Key ideas: An object is put onto a page by tapping on its icon on the Tool Menu. Then, you can interact with it in three different modes—Select, Draw, or Erase—found on the Tool Menu.
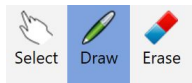
- **Select Mode** allows selection and moving of objects. Tap on an object to bring up action buttons (**delete**, **resize**) and a sub-menu for other actions that are unique for each kind of object.
- **Draw Mode** allows drawing and writing on the page or over an object with ink. Select from pen, marker or highlighter, and various colors.
- **Erase Mode** allows erasing of any ink on the page. If the top of your tablet pen has an eraser, you also can use that to erase ink.

Put a tool on the page Tap the tool icon on the Tool Menu.	
Bring up action buttons **Select Mode:** Tap on the tool. 1. **Delete button**. Tap on it to delete. 2. **Resize button**. Tap and drag on it to resize.	
Bring up tool menu **Select Mode:** Tap on the tool. Every tool's menu will be different.	

Array

Mathematical Purpose

The array tool supports development and extends understanding of multiplication and division by:

- Providing a visual model of multiplication (each dimension is a factor, the total number of squares is the product)
- Providing a visual model of division (the total number of squares is the dividend, one factor is the divisor and one is the quotient)
- Demonstrating the inverse relationship between multiplication and division
- Enabling students to build upon familiar number relationships to construct new ones, e.g., by combining smaller arrays representing known number facts to create larger arrays

A variant of this tool can be used to facilitate algebraic reasoning, in particular, generalization represented by an array dimension of an undetermined number, "N." Watch the video at the Technology for Mathematical Argumentation website to learn more: tma.mit.edu

Function

Users can:

- Create arrays by specifying the number of rows and the number of columns
- Annotate arrays (e.g., draw on them to demonstrate subdividing, record equations, help count squares)
- Manipulate arrays (e.g., rotate, move, break into smaller components and reassemble, resize)

Examples

Multiplication with skip counting strategy

Multiplication with partial products strategy, using array divide tool or dividing with ink

Hamid is putting chairs in rows.
He puts 8 chairs in each row. He makes 9 rows.
How many chairs are in rows?

OR

$5 \times 8 = 40$

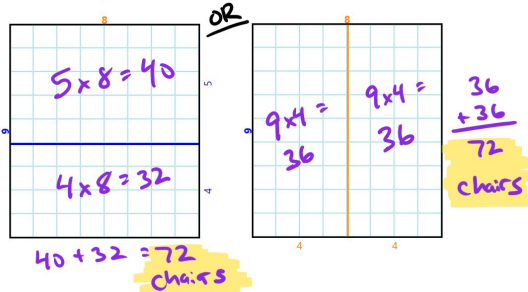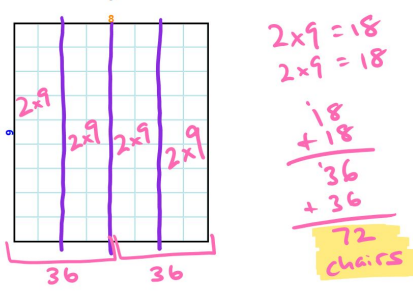$4 \times 8 = 32$

$40 + 32 = 72$ chairs

$9 \times 4 = 36$ $9 \times 4 = 36$

$\begin{array}{r} 36 \\ +36 \\ \hline 72 \end{array}$ chairs

Hamid is putting chairs in rows.
He puts 8 chairs in each row. He makes 9 rows.
How many chairs are in rows?

2×9 2×9 2×9 2×9

36   36

$2 \times 9 = 18$
$2 \times 9 = 18$

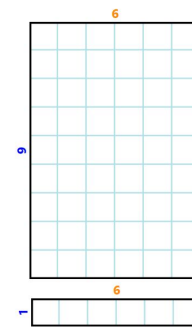$\begin{array}{r} 18 \\ +18 \\ \hline 36 \\ +36 \\ \hline 72 \end{array}$ chairs

Multiplication with subtraction strategy, using array divide tool or array cut tool

Hamid is putting chairs in rows.
He puts 8 chairs in each row. He makes 9 rows.
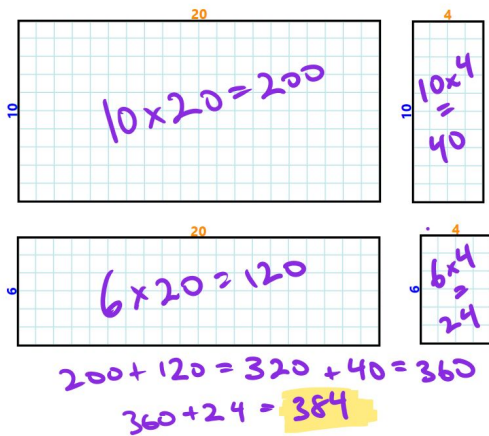How many chairs are in rows?

$10 \times 8 = 80$
$8 \times 1 = 8$
$80 - 8 = 72$ chairs
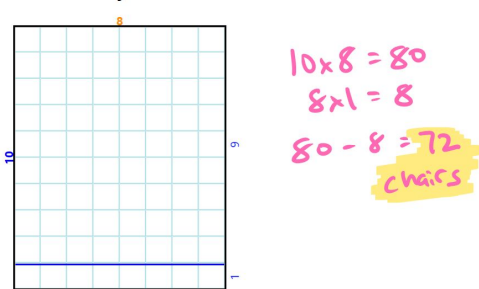
$9 \times 6 = $ _____

$10 \times 6 = 60$
$1 \times 6 = 6$
$60 - 6 = 54$

Multiplication with partial products strategy, using array cut tool

16x24

$10 \times 20 = 200$

$10 \times 4 = 40$

$6 \times 20 = 120$

$6 \times 4 = 24$
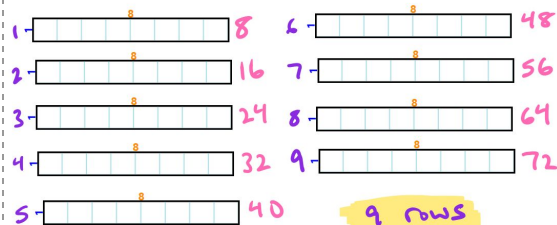
$200 + 120 = 320 + 40 = 360$
$360 + 24 = 384$

Division with skip counting and adding 1x arrays one at a time

Hailey is putting chairs in rows. She has 72 chairs.
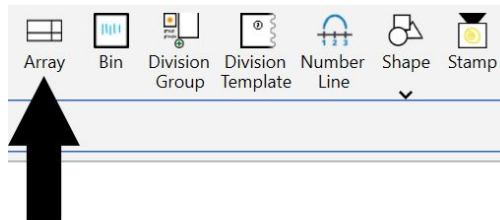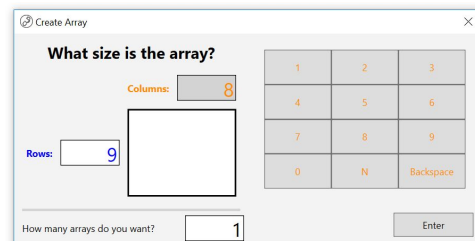She puts 8 chairs in each row.
How many rows does she make?

1- ⬜ 8

Hailey is putting chairs in rows. She has 72 chairs.
She puts 8 chairs in each row.
How many rows does she make?

1- ⬜ 8   6- ⬜ 48
2- ⬜ 16   7- ⬜ 56
3- ⬜ 24   8- ⬜ 64
4- ⬜ 32   9- ⬜ 72
5- ⬜ 40   9 rows
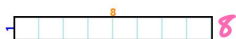
How to

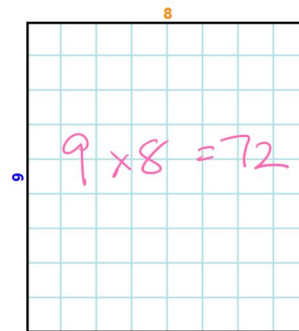|	
Put an array on the page Tap the **Array** icon on the Tool Menu.	Array · Bin · Division Group · Division Template · Number Line · Shape · Stamp
Determine the array dimensions and number of arrays ● Enter the numbers of rows and columns. ● For more than one of the same array, delete and change the "1" in the box following the text **"How many arrays do you want?"** Note: If you want an undetermined number of rows or columns, enter **N** for the row or column. You cannot have an undetermined number of *both* rows and columns.	Create Array What size is the array? Columns: 8 Rows: 9 How many arrays do you want? 1 1 2 3 / 4 5 6 / 7 8 9 / 0 N Backspace / Enter
Position or modify the array **Select Mode:** Tap on the array and drag to move the array with your finger or pen. Tapping on it also brings up the action buttons to **Resize** and **Delete**. The Array Menu appears when you tap on the array. Features on the menu: ● **Make Copies** ● **Rotate** (array by 90 degrees) ● **Show/Hide Labels** (array dimensions)	✖ Delete · ⊕ Make Copies · ◇ Rotate · Hide Labels · Hide Grid Lines · ⊕ Snap 8 9
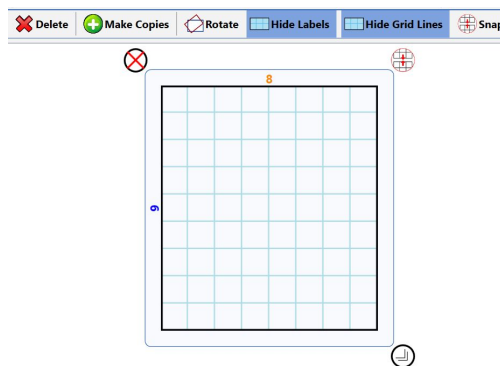
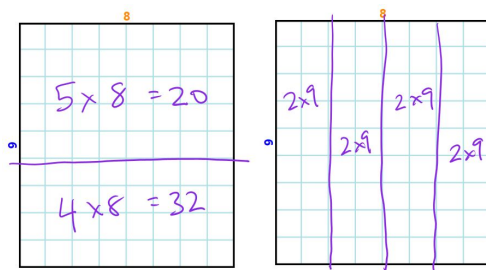|	
● **Show/Hide Grid Lines** (on the array)	
## Annotate the array **Draw Mode:** Write equations or numbers directly on top of, or beside, the array. <u>Note</u>: Any annotations on top of the array will be moved if the array is moved, and deleted if the array is deleted.	
## Subdivide array (Option 1: Draw divider) **Draw Mode:** Draw a line or multiple lines across or down the array. <u>Note</u>: If you move the array, the lines you drew over the array will move with it.	
## Subdivide array (Option 2: Add divider) 1. Select **Add Divider** on the Tool Menu. You'll be in **Select Mode**. Stay in **Select Mode** and draw a line across or down the array where you want the divider. 2. Question marks (**?**) will appear for you to enter the new array dimensions. Tap on question mark to enter the dimensions of the subparts of the array.	
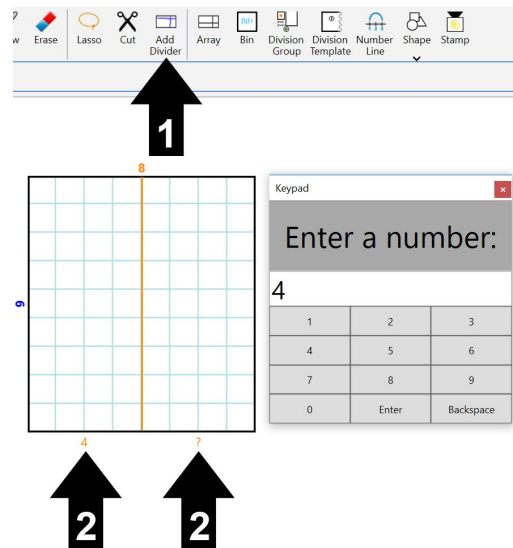
Subdivide array (Option 3: Cut the array)

1. Select **Cut** on the Tool Menu. You'll be in **Select Mode**. Stay in **Select Mode** and draw a line across or down the array to cut it into two new arrays.
2. The dimensions on the new arrays will be labeled automatically. You can move the arrays apart.
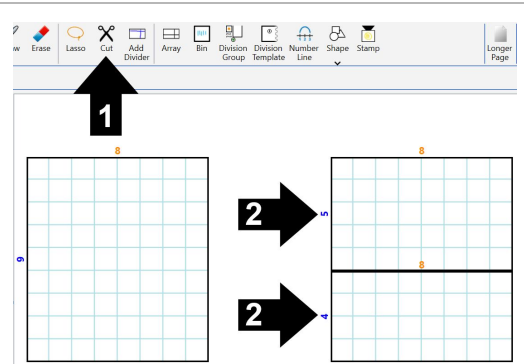
Combine two arrays with a dimension in common

Bring two arrays side by side or one on top of the other, with the common dimension aligned. Tap on either array, then tap on the **Snap action button** or **Snap** on the Array Menu. A single array with new dimensions results.
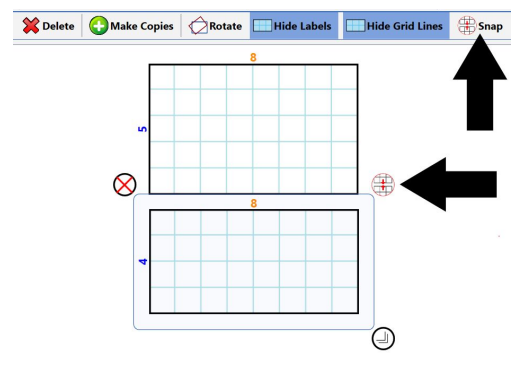
Arrays with one dimension labeled as N

Used to facilitate algebraic reasoning. Watch the video at the **Technology for Mathematical Argumentation (TMA)** website to learn more: tma.mit.edu
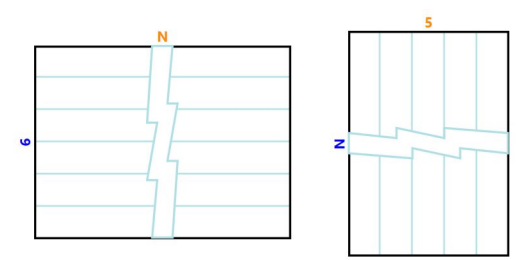
Bin

Mathematical Purpose

The Bin tool supports understanding of division by providing a visual model of the dividend (total number of tally marks), divisor, and quotient.

Function

Users:

1. put down bins
2. fill bins with tally marks

As users fill bins with tally marks, the tool automatically displays the:

- total number of tally marks in bins
- number of tally marks in each bin
- number of bins

Examples

Partitive division

Put down the same number of bins for the number of groups needed, then deal out tally marks in each bin, while maintaining the same number in each bin.
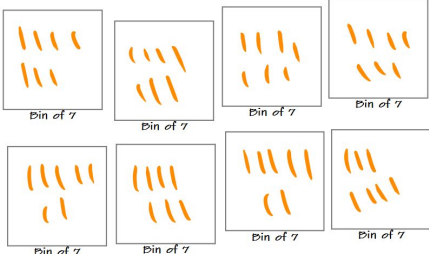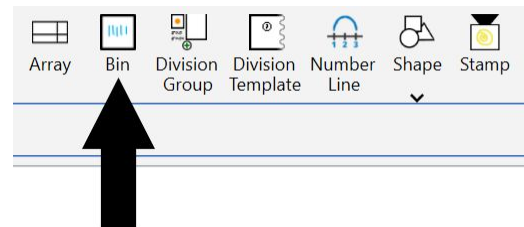
Quotative division

Put down bins and then fill one bin at a time with same number (size of group). Add more bins as needed.

Malik has 56 cookies.
He gave 7 cookies to each cousin.
How many cousins got cookies?

14 in Bins
3 Bins

Bin of 7   Bin of 7   Bin of 0

Malik has 56 cookies.
He gave 7 cookies to each cousin.
How many cousins got cookies?

56 in Bins
8 Bins

Bin of 7   Bin of 7   Bin of 7   Bin of 7
Bin of 7   Bin of 7   Bin of 7   Bin of 7

How-to

|	
Put a bin on the page Tap the **Bin** icon on the Tool Menu. Each time you tap, an empty bin appears on the screen.	
Position and size the bin(s) **Select Mode:** Move the bin with your finger or pen. Tap on it to bring up the action buttons to **Resize** and **Delete**. The Bin Menu also appears when you tap on a bin.	
Fill the bin(s) with tally marks **Select Mode:** Draw tally marks in the bin(s). Each pen stroke counts as one mark.	
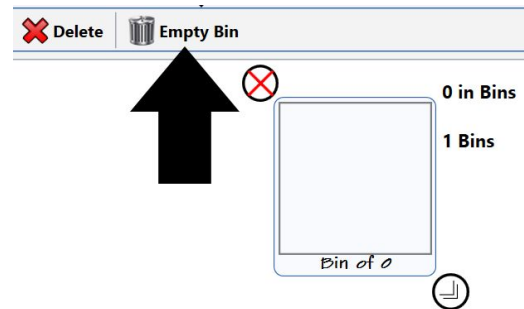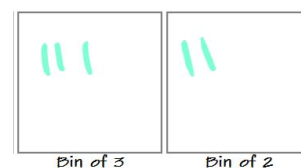
Keep track of numbers

These numbers automatically change each time a tally mark is added to a bin.

- The number of tally marks in bins and the number of bins.
- The number in each bin, or size of each bin

56 in Bins

8 Bins

Bin of 7

1

2
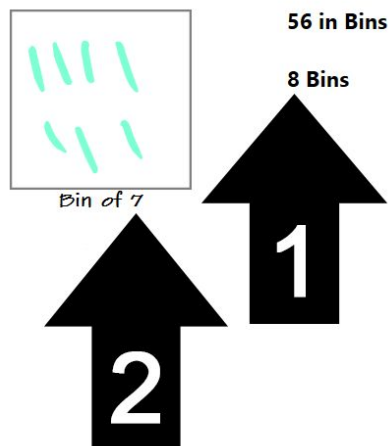
Clear the bin of tally marks

Select Mode: Tap on the bin to bring up the Bin Menu. Click on Empty Bin to clear the bin.

<u>Note</u>: This clears the *selected* bin, not *all* of the bins on the page.

Delete   Empty Bin

Bin of 7
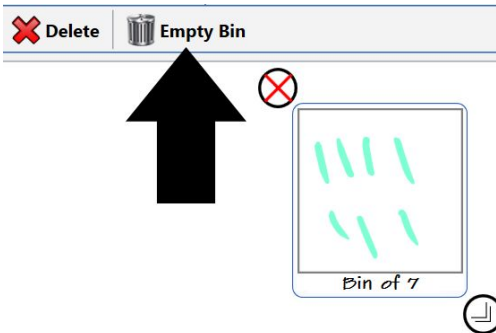
Division Group

Mathematical Purpose

This tool supports understanding of division by providing a visual model of the dividend (total number of objects), divisor, and quotient.

Function

Users:

1. create an image for an "object" and an image for a "group"
2. make copies of "object" and "group"
3. move objects into groups

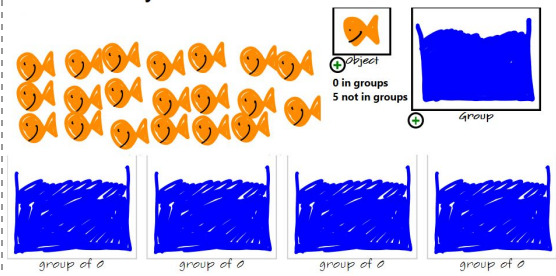As users move objects into groups, the tool displays the:

- total number of objects that have been moved into groups
- total number of objects that have not (yet) been moved into groups
- number of objects in each group

Examples

Partitive division

Make appropriate number of groups, then fill by dealing out objects, maintaining the same number of objects in each group.
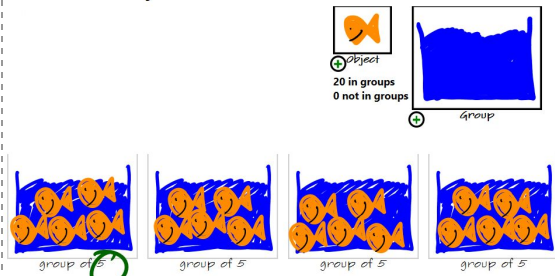
Quotative division

Make and fill one group at a time with the appropriate group size

Paola has 20 fish in fish tanks.
Each tank has 5 fish in it.
How many tanks are there?

Object
5 in groups
15 not in groups
Group

group of 5   group of 0

Paola has 20 fish in fish tanks.
Each tank has 5 fish in it.
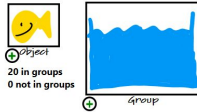How many tanks are there?

4 tanks

Object
20 in groups
0 not in groups
Group

group of 5   group of 5   group of 5   group of 5

How-to

Put division group makers on the page

Tap the **Division Group** icon on the Tool Menu.

Array   Bin   Division   Division   Number   Shape   Stamp
   Group   Template   Line
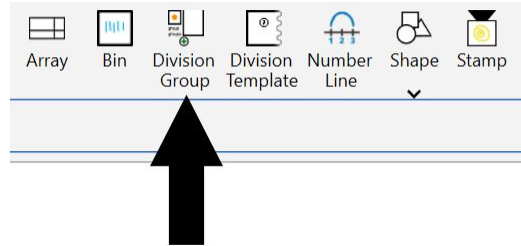
Position and size the division group makers so you have enough space to draw your images

Select Mode: Move the makers with your finger or pen. Tap on them to bring up the action buttons to **Resize** and **Delete**.

Note: If you delete one of the makers, the other maker also is deleted.

Note: The Object Maker Menu and Group Maker Menu also appear when tapping on either maker.

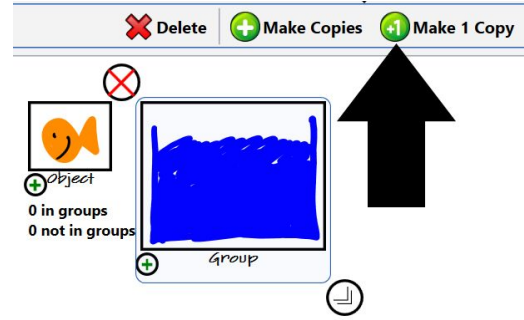Delete   Make Copies   Make 1 Copy

object
0 in groups
0 not in groups
Group
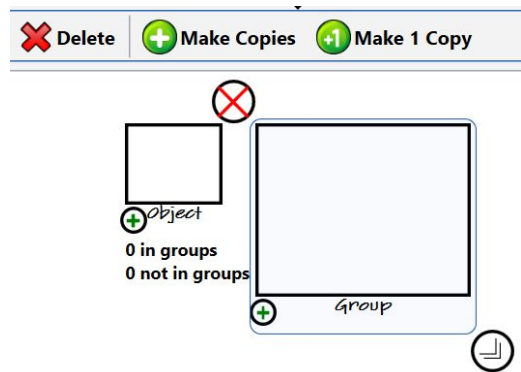
Create your object and group images

Draw Mode: Draw images inside the object maker and group maker.
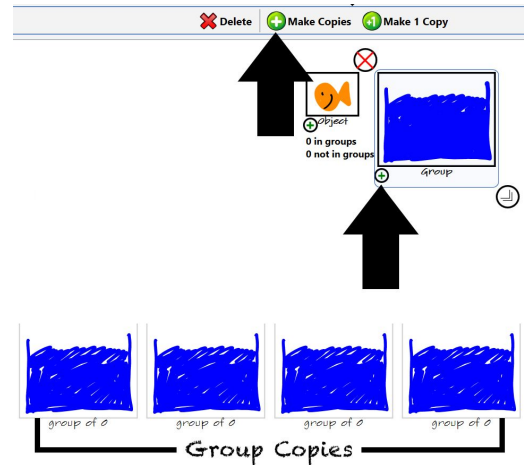
object
0 in groups
0 not in groups
Group

Make copies of group
(Option 1: One at a time)

Select Mode: Tap on **Make 1 Copy**.

## Make copies of group
(Option 2: Several at once)

Select Mode: Tap on the plus sign (**+**) underneath the Group Maker, or tap on **Make Copies** on the Group Maker Menu. Enter the number of groups you want. Groups will appear all at once on the page.

Make copies of object

Select Mode: Tap on the plus sign (**+**) underneath the object maker, or tap on **Make Copies** on the Object Maker Menu. Enter the number of objects you want. Objects will appear all at once on the page.
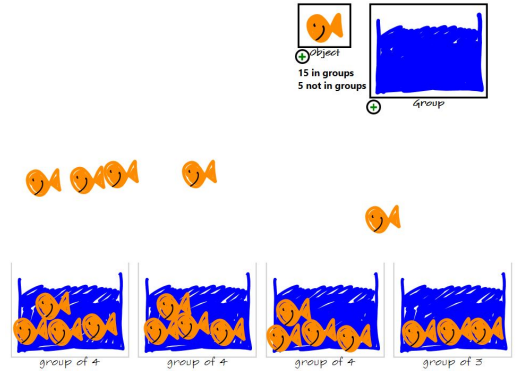
Note: You can move the object copies one at a time, or many at once with **Lasso** on the Tool Menu.
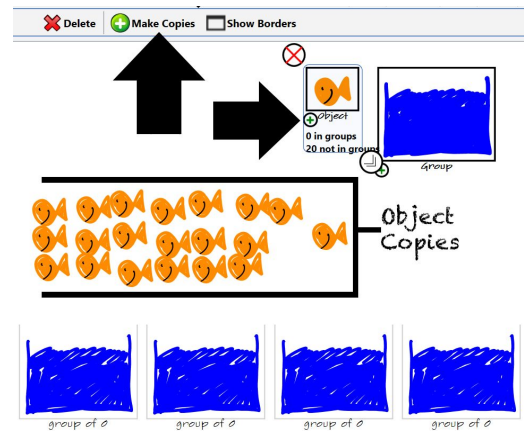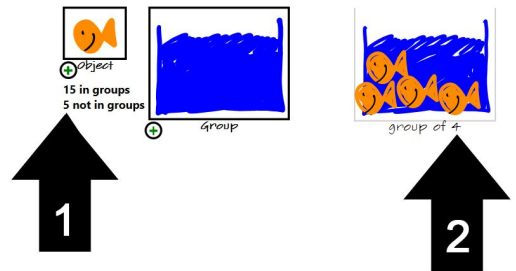
Place objects into groups

Move objects on top of the groups one by one.

Keep track of numbers

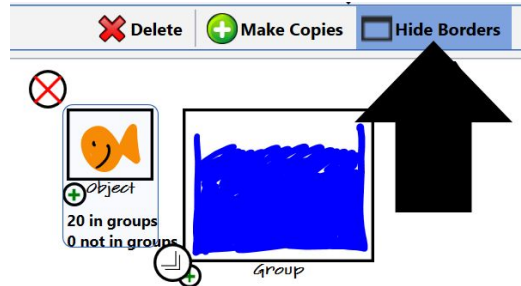These numbers change each time an object is moved into a group.

1. The numbers of objects "in a group" and "not in a group"
2. The number in each group, or size of each group

Object Maker Menu: Additional feature

Select Mode: Tap on the object maker to bring up its menu.

*   **Show/Hide Border** around the Objects.

Delete an object or group

Delete the same ways you would an object or group maker (two ways to do this):

Select Mode: Tap on the object maker to bring up its action buttons and menu. Use the:

*   **Delete action button**.
*   **Delete** button on the Object Maker Menu.

Note: If you delete a group with objects inside it, the objects inside the group also will be deleted.
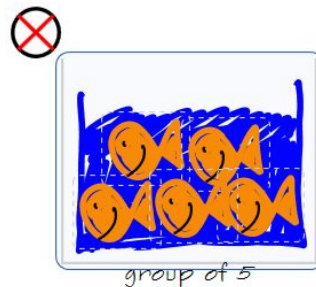
Division Template

Mathematical Purpose

This tool enables those already familiar with the array model for multiplication to explore division and relationships between multiplication and division. The dividend is represented by the total number of "tiles," or squares, with which to fill an array; the divisor is represented by one dimension of an array. The array is shown as a template (with a jagged line along one dimension), so the quotient is not revealed. To determine the quotient, the user fills the array with component arrays with the divisor as one dimension. The tool tracks the accumulating dimensions and the remainder.

Function

Users can:
- Create a division template by specifying the dividend and divisor
- Fill the division template with component arrays
- Option to display tiles representing the dividend; the tiles can then be grouped into an array, which is inserted into the division template
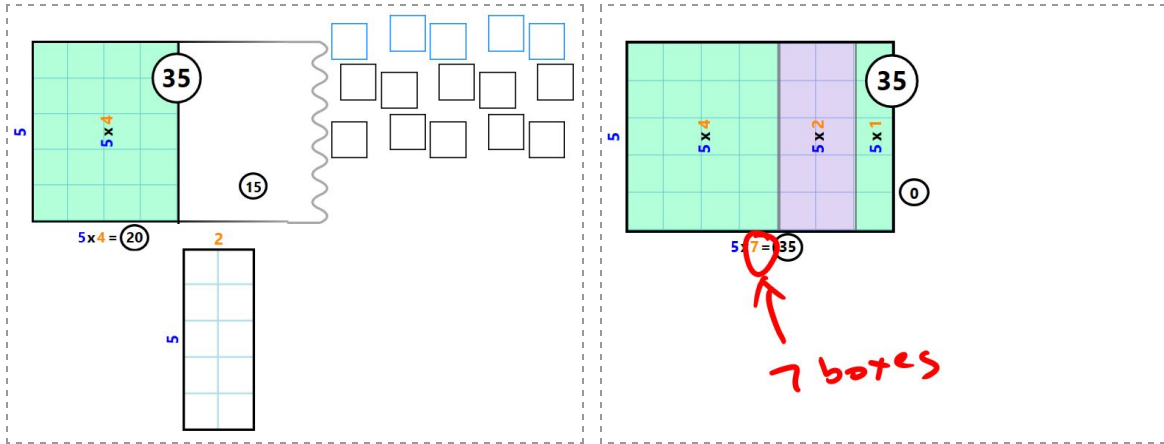
As users move arrays into the division template, the tool displays:
- total number of tiles that have been inserted into the array
- number of groups (of the divisor) inserted so far
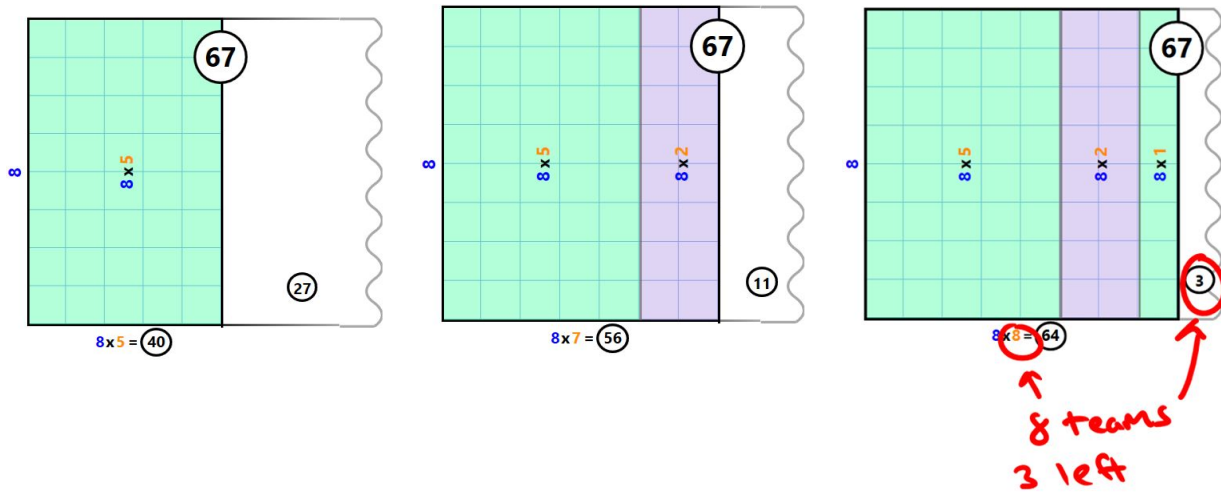- tiles left to be inserted

Examples

Dina has 35 oranges. She puts 5 oranges in each box. How many boxes does she have?

35

5 × 4

15

5 × 4 = 20   2

5

35

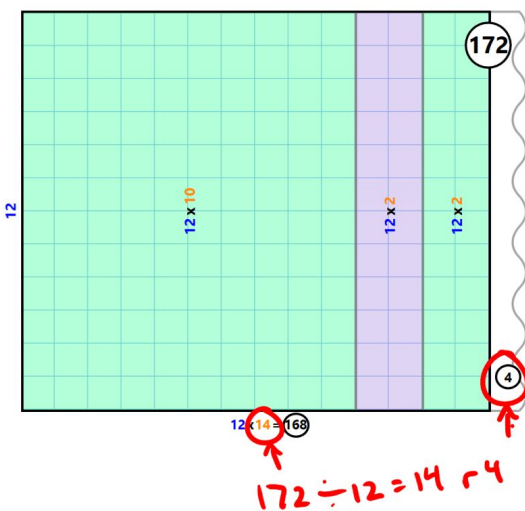5 × 4   5 × 2   5 × 1

0

5 × 7 = 35

↑ 7 boxes
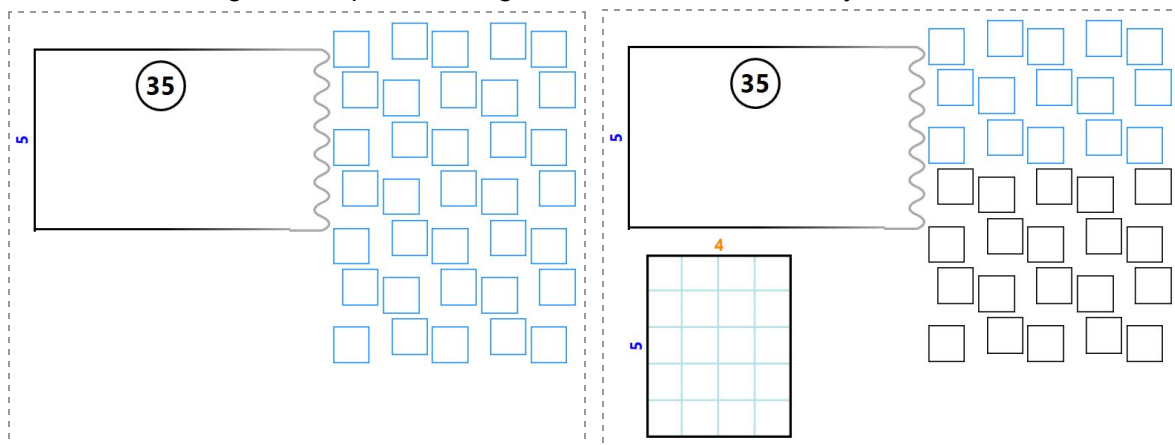
67 students need to form teams for a gym game. Each team has 8 people.
How many teams can they make? Can everyone be on a team? (If not, how many are left out?)
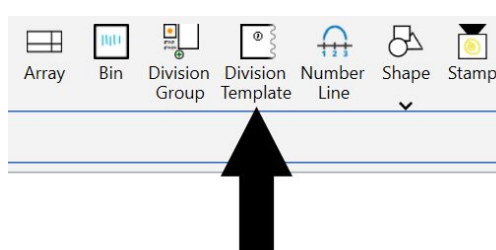

67

8 × 5

27

8 × 5 = 40

67

8 × 5   8 × 2

11

8 × 7 = 56

67

8 × 5   8 × 2   8 × 1

3

8 × 8 = 64

↑   ↑
8 teams
3 left

172 ÷ 12

172

12 × 10   12 × 2   12 × 2

4

12 × 14 = 168

172 ÷ 12 = 14 r 4

How to

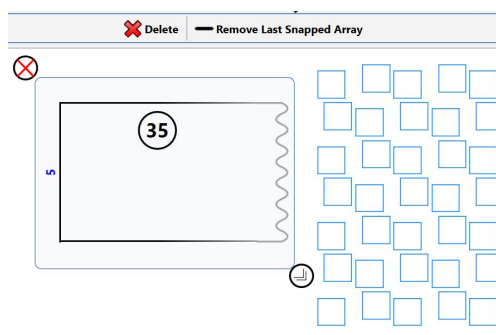|	
Put a division template on the page Tap the **Division Template** icon on the Tool Menu.	
Determine the dividend and divisor Enter the dividend (product) and divisor (factor). Check the box next to **"Show tiles?"** if you want a visual representation of the 1x1 arrays that need to fit into the template. Note: Tiles do not appear for dividends larger than 50.	
Position and change the division template **Select Mode:** Move the division template with your finger or pen. Tap on it to bring up the action buttons to **Resize** and **Delete**. The Division Template Menu also appears when you tap on the template. You can also delete from the menu. Note: If you delete the division template, any snapped in arrays will also be deleted.	
Snap in arrays Tap the **Array** icon on the Tool Menu. Determine the array dimensions (the number of rows should match the known factor, or divisor, on the division template). **Select Mode:** Move the array with your finger or pen to place it over the division template and line up the common dimension. Make sure the common dimension is the number of rows, not columns. Otherwise, **Rotate** the array by clicking the button on the Array Menu. Then snap the array into the division template with the **Snap action button** or **Snap** on the Array Menu.	
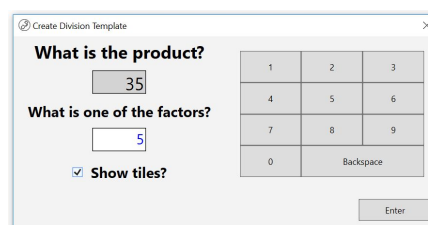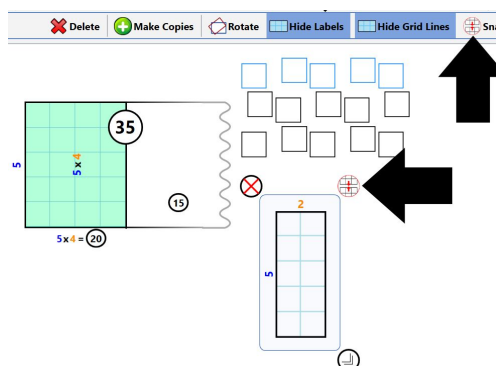
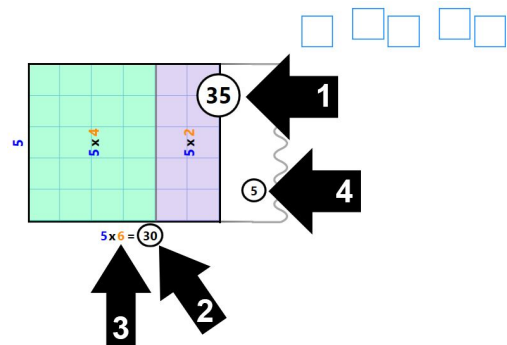|	
Note: When the division template is full, the border will flash red if you try to snap in another array, and the array will not snap in. Any tiles left constitute the remainder. Note: If you resized the division template or array, they don't need to be the same size; the array will snap in if the common dimension is aligned.	
## Keep track of numbers 1. The product (dividend) of template. These numbers change each time an array is snapped into the division template: 2. Total size (product or dividend) of the snapped in arrays 3. Total number of columns (quotient) 4. Remainder	
## Remove last snapped array from division template **Select Mode:** Tap the division template. Tap on **Remove Last Snapped Array** on the Division Template Menu. Note: You can continue to remove the last snapped array until there are no more arrays left in the division template.	
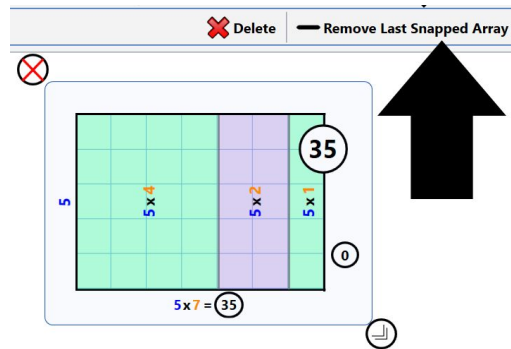
Number Line

Mathematical Purpose

This tool provides a visual model of multiplication and division, as users create equal-sized arcs starting at zero and extending through a skip-counting sequence (e.g., 0, 5, 10, 15...)

- For multiplication, the arc size represents one factor, the number of arcs represents the other, and the endpoint of the sequence of arcs reflects the product;
- For division, the endpoint of the sequence of arcs represents the dividend, the arc size reflects the divisor, and the number of arcs of that size needed to reach the dividend is the quotient.

Function

Users can:

- Create a number line by specifying the length (or endpoint) of the number line
- Create arcs (two ways)
  - Draw arcs from one number to another
  - If users selected "automatic arcs," the tool will create the arcs as the user marks the start and end points for the jumps.

The tool automatically displays the length of the arc over each arc a user creates.

Examples

Kyla has 8 bookshelves. Each bookshelf holds 6 books.
How many books can her bookshelves hold?

Multiplication

If the length initially chosen for the number line isn't long enough to accommodate the needed number of arcs, the number line can be lengthened.
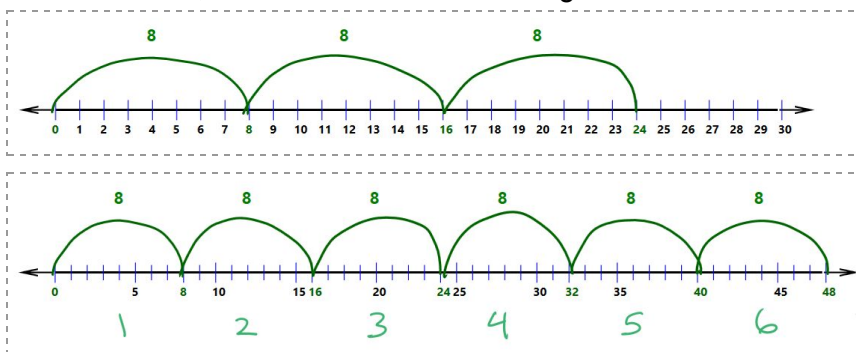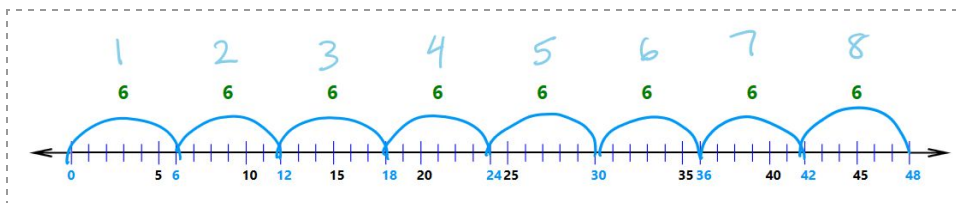
Multiplication

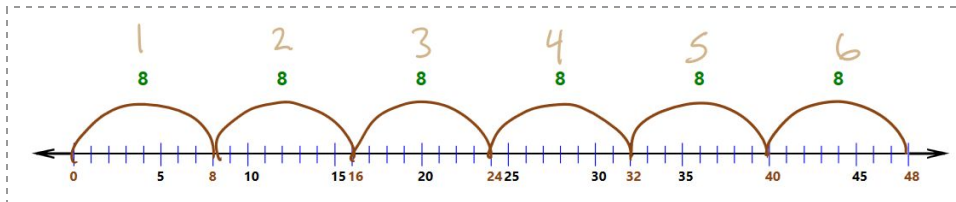The number line may be longer than the answer.
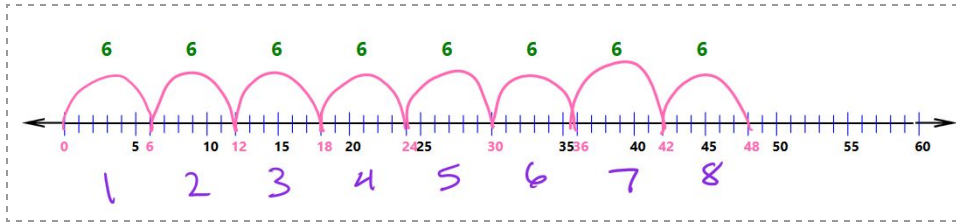
Quotative Division

Kenji has 48 books. He puts 6 books on each bookshelf. How many bookshelves does he need to hold all his books?
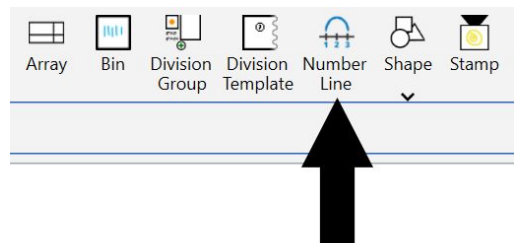

Partitive Division

Kelani has 48 books. She puts the books onto 8 bookshelves. Each bookshelf holds the same number of books. How many books are on each bookshelf?

How to

Put a number line (NL) on the page

Tap the **Number Line** icon on the Tool Menu.

Determine the length of NL

Enter the length (endpoint) of the NL.
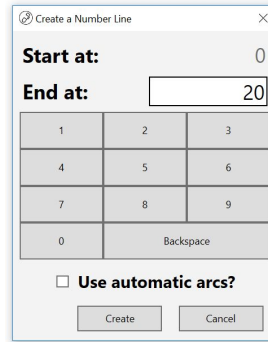
Note: Number lines cannot be longer than 84. For problems with products or dividends larger than 84, use the **Division Template** or **Bins**.

Note: When a NL is the length of 30 or less, it will display numbers incrementally by 1. When the NL is longer than 30, it will display numbers incrementally by 5.

## Determine the type of NL (two types)

Check the box next to **Use automatic arcs?** to get an auto arcs NL, or leave it unchecked to get the regular NL.

1. Regular NL: draw your own jumps or arcs.
2. Auto arcs NL: computer draws jumps or arcs. Useful when the NL gets long, making the tic marks very close together, and for students for whom drawing the arcs may be difficult.
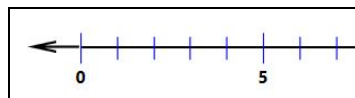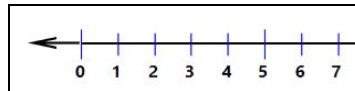
Position and size the NL

Select Mode Move the NL with your finger or pen. Tap on it to bring up the action buttons to **Resize** and **Delete**. The NL Menu also appears when you tap on the number line.
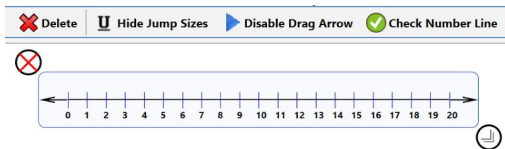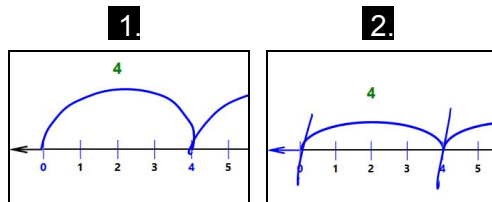

Make jumps on the NL

Draw Mode:

1. Regular NL: Draw an arc from one number to another to create a jump. Lift the pen off the screen after each arc.
2. Auto arcs NL: Mark the start and end numbers with a short line or slash across the tic marks. The computer will create an arc connecting the two numbers.

Note: The NL tool will indicate the start and end numbers of each jump on the NL by changing the black color to the same color that the jumps are made in. It will also insert the jump size above each arc.
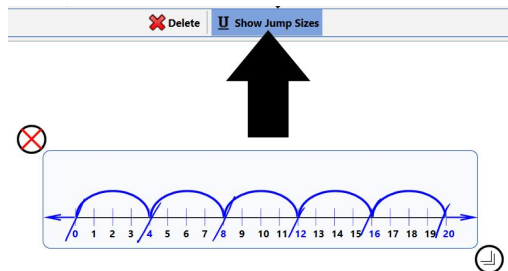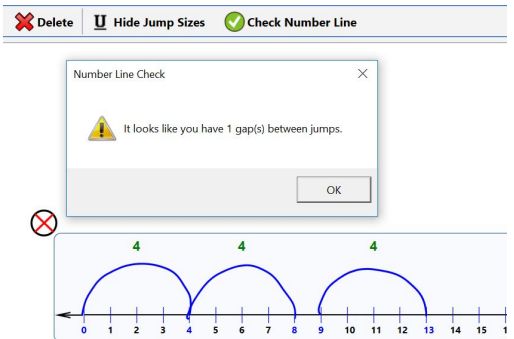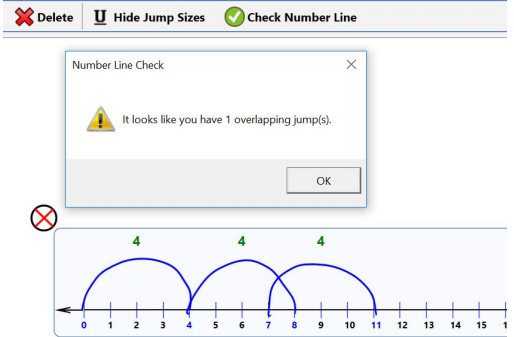
| **Show/Hide Jump Sizes** using the NL menu.	
## Check jumps on the NL This feature is available with the non-automatic arcs number line. It tells you when there are overlaps in jumps or gaps between jumps. **Select Mode:** Tap on the NL. Tap on **Check Number Line** on the NL Menu.	
## Change length of NL 1. **Select Mode:** Tap on the NL. Tap on the end (right) arrow on the NL. Enter the new endpoint. 2. If you change the length of a NL with jumps on it, the jumps will be resized automatically.	
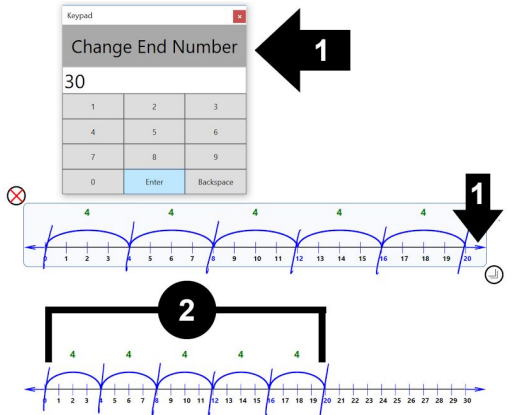
Stamp

Mathematical Purpose

This tool supports understanding of multiplication by providing a visual model of the multiplier (number of groups), multiplicand (number in each group), and product. The representation offers scaffolding for using skip counting or repeated addition to find the product.

Function
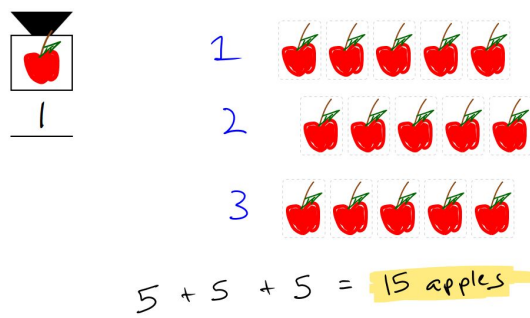
Users:

1. create an image using the Stamp Tool to show a single item or a group of items
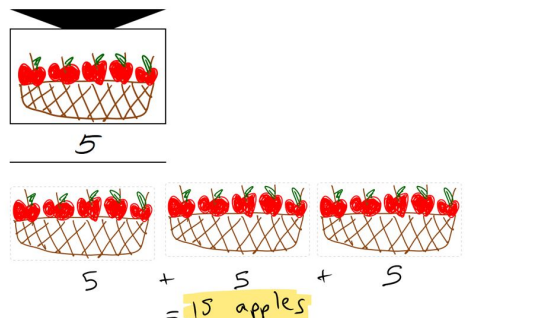2. label the stamp with the number of items it represents
3. make copies using the Stamp Tool

Examples

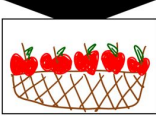### Discrete objects: Individual stamp and group stamp

Ana has 3 baskets.
Each basket has 5 apples in it.
How many apples are in Ana's baskets?

1
2
3

5 + 5 + 5 = 15 apples

Ana has 3 baskets.
Each basket has 5 apples in it.
How many apples are in Ana's baskets?

5

5 + 5 + 5
= 15 apples

Unitized objects

Alex has 6 pet turtles. Each turtle has 4 legs.
She also has 4 pet spiders. Each spider has 8 legs.
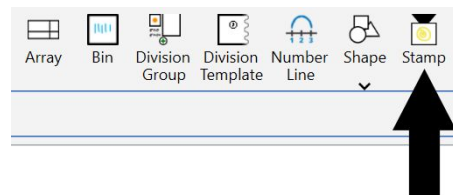How many legs are there in all?

How to

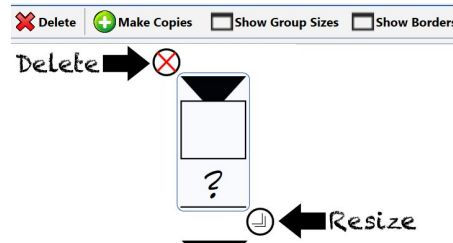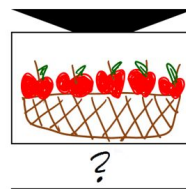|	
Put a stamp tool on the page Tap the **Stamp** icon on the Tool Menu.	
Position and size the stamp Tool Resize the stamp tool so you have enough space to draw your image. **Select Mode:** Move the stamp tool with your finger or pen. Tap on it to bring up the action buttons to **Resize** and **Delete**. Note: The Stamp Menu also appears when you tap on the stamp tool.	
Create your stamp **Draw Mode:** Draw an image inside the stamp tool.	
Record group size **Select Mode:** Tap the question mark (**?**) and enter the number of items your stamp represents.	
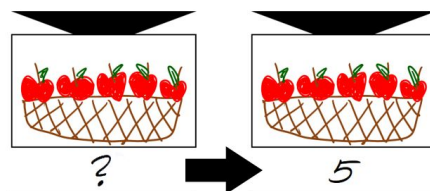
Make copies (Option 1: One at a time)

Select Mode: Place your pen or finger on the stamp handle, drag to the location where you want a copy, then release. Repeat until you have the number of copies you want.

Note: The stamp tool itself does NOT count as one of the copies.
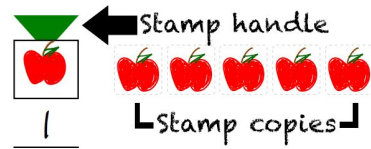
Make copies (Option 2: Several at once)

Select Mode: Tap on the stamp tool to bring up the Stamp Menu. Then tap on **Create Copies,** and enter the number you want. Your specified number of stamps will appear automatically. You can move the stamps one at a time, or many at once with **Lasso** on the Tool Menu.
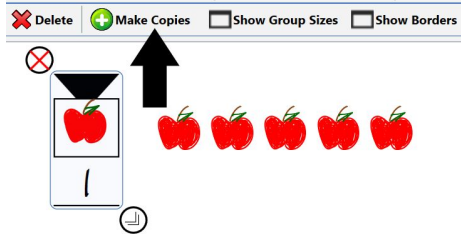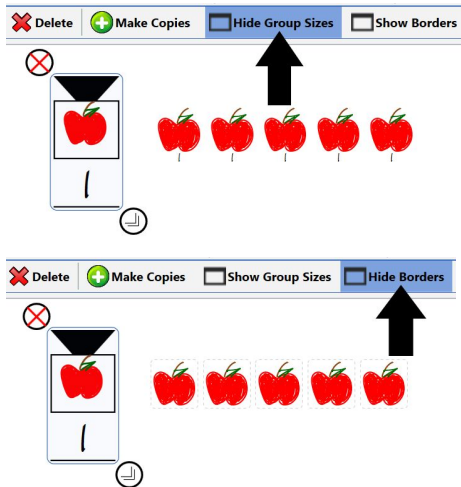

Stamp Menu and Stamp Copy Menu features

Select Mode: Tap on the stamp tool or stamp copy to bring up its menu.

- **Show/Hide Group Size(s)** for all or singular stamp copies (toggle). The group size appears below the copy.
  ProTip: This may help students with repeat addition.
- **Show/Hide Border** around all or singular stamp copies (toggle).

Delete a Stamp Copy

Delete the same ways you would a stamp tool (two ways to do this):

Select Mode Tap on the stamp copy to bring up its action button and menu. Use the:

- **Delete action button**.
- **Delete** button on the Stamp Copy Menu.